JOURNAL OF AIRCRAFT
Vol. 36, No. 1, January–February 1999

Recent Applications of Design Optimization to Rotorcraft—A Survey

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Introduction

THE successful design of a helicopter relies, perhaps to a greater extent than for any other aerospace vehicle, on the tight integration of a variety of aeronautical engineering disciplines. An example of this is the design of the main rotor system. Rotor blades are slender, flexible beams. Even in normal operating conditions they may undergo elastic deformations in bending and torsion that can be beyond the limits of linear beam theories and, therefore, moderately large deformations need to be taken into account. Blade flexibility and related dynamic effects influence not only blade and hub stresses and fatigue life, but also interact with the aerodynamic loading, because the deformations of the blades substantially modify the effective angle of attack of each cross section. In turn, rotor-blade aerodynamics is coupled with structural dynamics because much of the damping in flap and lag bending and in torsion is of aerodynamic origin. With the partial exception of hovering flight, the structural and aerodynamic problems of a helicopter rotor blade are intrinsically unsteady. Even in steady cruise conditions, the blade can encounter airflow velocities that range from transonic or slightly supersonic to low speed, including stall, and reversed flow. The variations occur with a period of the order of 200 ms. Therefore, the calculation of aerodynamic forces and structural loads on a rotor blade is really an integrated aeroelastic problem.

An even broader integration may be required when handling qualities considerations are brought into the design. Modern flight-control systems can modify the natural characteristics of the response of the helicopter to pilot inputs, in ways that make the helicopter easier to fly and reduce the pilot workload. For example, a flight-control system can make the helicopter react to a step input of lateral control with a step variation of roll angle, instead of the more natural step variation of roll *rate*. Experience has shown that the ability to achieve this kind of *tailored* handling quality characteristics can be limited by adverse dynamic interactions with the main rotor system. This way, the disciplines of flight dynamics and control-system design become closely coupled with structural dynamics and aerodynamics.

These are but two of the many examples of helicopter engineering problems that require a multidisciplinary approach. This requirement has motivated the development of *comprehensive* analysis codes during the last two decades. Such codes attempt to integrate, in a single software program, the mathematical models required by each helicopter engineering discipline.[1–6] It is beyond the scope of the present paper to describe these codes in any detail. They are mentioned here as evidence that the design of a helicopter is intrinsically multidisciplinary, and perhaps more so than for any other aerospace vehicle.

Design optimization appears to be an ideal methodology to help simplify helicopter design because it can provide a systematic way of integrating the various disciplines involved. However, despite considerable activity in this field, helicopter optimization has not yet reached the same maturity and acceptance as structural optimization. Therefore, the main objectives of the present paper are not only to review the current state of the art for rotorcraft applications of optimization, but also to identify issues that have impeded its progress.

An extensive survey of multidisciplinary design optimization (MDO) applications to aerospace design has been recently presented by Sobieszczanski-Sobieski and Haftka.[7] The present paper is meant to supplement it with a narrower focus on rotorcraft. Research in helicopter applications of optimization started in earnest in the early 1980s with the pioneering works of Refs. 8–12. Extensive research in this area was carried out throughout the 1980s. This research is reviewed in two papers by Friedmann[13] and Adelman and Mantay[14]; the first focuses on the use of optimization for vibration reduction, whereas the second addresses MDO applications. The present paper will focus primarily on developments in subsequent years.

The words *helicopter* and *rotorcraft* will be used interchangeably. In fact, with the notable exception of tilt-rotor and tilt-wing aircraft, the rotorcraft of primary interest for optimization applications is the helicopter. Many practical helicopter optimization problems are not particularly different from those of other aerospace or ground vehicles. An example could be the minimum weight, structural optimization of a fuselage subject to preassigned harmonic loads, and with stress constraints. Therefore, the present paper will only address those design problems that are unique to the helicopter; in most cases this implies that the presence of the main rotor system is a dominant ingredient in the optimization.

In general, no attempt will be made to extract specific design guidelines from the literature reviewed, such as, for example, ply orientations, airfoil shapes, or planform geometries of rotor blades. As pointed out later in this paper, many practical helicopter problems still cannot be modeled with the desired accuracy, and this affects negatively the reliability of the results obtained from optimization.

The majority of the helicopter optimization studies have addressed some form of the rotor dynamic behavior, such as

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Received April 1, 1998; revision received Aug. 12, 1998; accepted for publication Aug. 12, 1998.

aeroelastic stability or vibratory loads. Studies in other areas will also be reviewed in the present paper; however, unless specified otherwise, comments and conclusions will usually refer to dynamics applications. In many helicopter applications, the boundary between uni- and multidisciplinary optimization is not well defined, and there is a lack of consensus within the helicopter community on this issue. What is a unidisciplinary aeroelastic optimization to some researchers may be a multidisciplinary structural/dynamic/aerodynamic optimization to others. This paper takes the point of view that such distinction is mostly a matter of semantics, and is not too critical in practice.

Challenges for the Applications of Design Optimization to Rotorcraft Problems

The application of optimization techniques to structural design has been quite successful. A good argument can be made that structural optimization is now a reasonably mature technology, to the point that optimization has been added to most major commercial finite element computer programs.[15] This success can be attributed to three key elements: 1) the availability of accurate analyses, 2) the availability of efficient sensitivity analysis techniques, and 3) the ability to generate accurate, mathematically simple approximations to many objective functions and behavior constraints of practical interests, including the availability of suitable intermediate design variables that extend considerably the range of validity of those approximations. Thus, many structural optimization problems of practical interest can be solved with no more than 10 complete analyses of the structure being designed. The success of structural optimization created the expectation that the same methodology could be applied directly to helicopter problems, and achieve the same level of success with similar computational efficiency. A decade of experience with this class of problems has revealed that such expectations were overly optimistic, and that considerable progress still remains to be made. The principal obstacle to success is the lack in the three key elements mentioned earlier. Each of the three elements is discussed next.

Availability of Accurate Analyses

In a 1985 survey of applications of optimization methods to helicopter design, Miura[16] stated that "One commonly expressed concern about the use of design-optimization methods in helicopter design applications is the availability of adequate analytical techniques." More than a decade later, and despite substantial progress in many fields, this concern persists. For example, the predictive capabilities of several flight dynamic simulation models have been evaluated by the European research working group Garteur AG06.[17] The results show that several areas of weakness remain, such as the underprediction of torque and power at high speed and the poor prediction of cross-coupling response, e.g., the roll response to longitudinal (pitch) pilot inputs. Another area of great importance in helicopter design is that of vibratory rotor loads. Reference 18 reports the results of a workshop on the correlation of vibratory load predictions from several advanced aeroelastic codes, compared with actual flight measurements. One of the conclusions of the study is that "On average, codes are not able to predict vibratory loads to an accuracy any greater than 50% of the [helicopter] measured loads," and also that "In general, the wider incorporation of more advanced [aerodynamic models] produces a minor enhancement to the overall group correlation standard."[18] Figure 1 shows an example of results from Ref. 18. The figure compares flight test results and seven different predictions of the 4/rev vibratory loads in the cockpit at two different speeds. The predictions with simple momentum inflow models appear to be quite poor. Noticeable improvements are obtained by including more sophisticated vortex wake models. Indeed, the complexity of the aerodynamic flowfield is an important reason of the difficulty in predicting vibratory

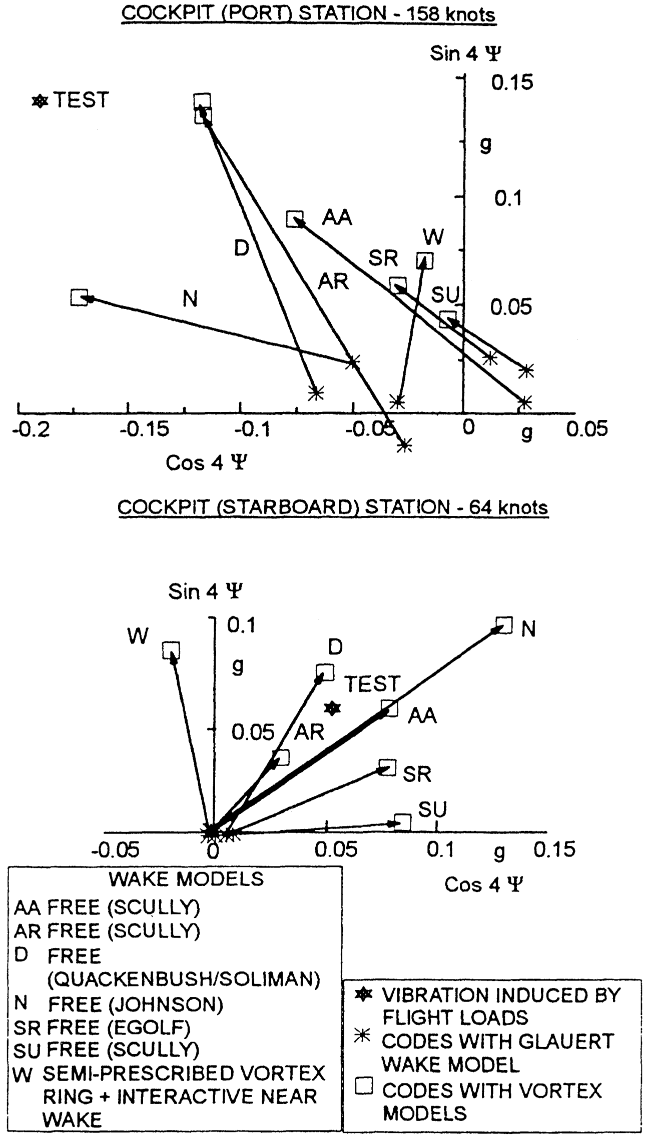

Fig. 1 4/rev fixed frame vibration vector changes caused by the inclusion of improved wake models, compared with Glauert- (momentum-) type model as baseline. Figure taken from Ref. 18.

loads. Reference 19 presents a review of the problems associated with the understanding of the vortex dynamics of helicopter rotor wakes. Vortex dynamics is responsible for rotor noise and vibratory loads for both rotor blades and fuselage. The authors state that "While these vortex related phenomena have been the subject of much research, they are still poorly understood because they are both difficult to measure as well as to predict."[19]

If the analysis models are not sufficiently accurate, it is inevitable that the results of an optimization based on them be looked upon critically if not with skepticism. This can dampen the enthusiasm for optimization. The assessments of the improvements brought about by the use of optimization could also be called into question. Comparing the initial and the optimum designs to define percentage improvements must be done very carefully in any optimization problem, because if the initial design is not very good, the benefits of optimization will be overestimated. For a helicopter problem, the uncertainty will be further increased if the underlying analysis is not sufficiently accurate.

One of the ways in which Ref. 16 addressed the problem of inadequate predictive capabilities was by suggesting that the analysis programs be kept as modular as possible, so that they could be quickly updated as better analyses became available. This way the optimization program would work with the best technology available at the time. This suggestion remains valid today.

Availability of Efficient Sensitivity Analysis Techniques

Most practical optimization algorithms require the calculation of the gradients of objective function and behavior constraints; some also require the matrix of second derivatives, or Hessian. Gradients and Hessians can be calculated numerically, using finite difference approximations, but this can be computationally expensive because at least $N + 1$ function evaluations are required for the gradient of a function of N variables. These sensitivities can also be obtained analytically, or semianalytically, i.e., by first using analytical chain-rule differentiation and then using finite differences for some of the resulting terms; these methods sometimes provide the sensitivities for a fraction of the cost of one function evaluation (besides the baseline). Expressions for the sensitivities of the solutions of various types of mathematical problems are available.[20] These include systems of linear and nonlinear algebraic equations, eigenvalue problems, and systems of ordinary differential equations (ODEs).

Difficulties in Semianalytical Sensitivities

While many methods for semianalytical sensitivity calculations can be directly applied to many rotorcraft problems, some special issues may arise. For example, consider the following system of linear, time-invariant, homogeneous ODEs:

$$M\ddot{u} + C\dot{u} + Ku = 0 \tag{1}$$

The aeroelastic stability of a helicopter rotor in hover can be obtained from the complex eigenvalue problem associated with a system such as Eq. (1). Then M, C, and K are the rotor mass, damping, and stiffness matrices, respectively. If μ is a complex eigenvalue, u the corresponding eigenvector, and x a design variable, the first-order sensitivity of μ with respect to x is[20]

$$\frac{d\mu}{dx} = -\frac{\mu^2 u^T \dfrac{dM}{dx} u + \mu u^T \dfrac{dC}{dx} u + u^T \dfrac{dK}{dx} u}{2\mu u^T M u + u^T C u} \tag{2}$$

where μ, u, M, C, and K are already available from the baseline analysis. The advantage of using Eq. (2), instead of finite differences, to calculate the eigenvalue sensitivity depends on the cost of calculating the sensitivities dM/dx, dC/dx, and dK/dx of the system matrices. In structural optimization problems based on finite elements, these sensitivities can be inexpensive if the design variable x is a cross-sectional dimension or a section property. In fact, they can often be obtained through simple differentiations of the element matrices. Also, the analysis code will likely have a library of finite elements, for which such element-level sensitivities could be computed a priori once and reused as needed. In a helicopter problem the calculation of these sensitivities may not be as simple. For example, the damping matrix C and the stiffness matrix K will usually contain aerodynamic contributions that may require involved iterative calculations; therefore, it may not be possible to obtain their sensitivities through simple analytic differentiations, and finite difference approximations would have to be used. Even more importantly, to set up the calculations required for dM/dx, dC/dx, and dK/dx, it will be necessary to modify the analysis program, which could be a major practical drawback. Currently. there are a few design-oriented helicopter analysis codes that provide both the behavior quantities of interest and their sensitivities with respect to design parameters. All of these codes have been developed at universities.[21,22,24,46] No industrial codes of this type have been reported in the literature.

Difficulties Due to Problem Nonlinearity

Sensitivity analyses may also be complicated by the nonlinearity of many helicopter problems. Consider, for example, the sensitivity of the aeroelastic stability eigenvalues in forward flight. These eigenvalues are obtained by applying Floquet theory to a system of nonlinear ODEs with periodic coefficients, linearized about a steady-state periodic equilibrium position, and are called *characteristic exponents*.[25] The real part of the characteristic exponents indicates the stability of the system. The sensitivity of the real part ζ_i of the ith characteristic exponent with respect to the design parameter x is given by[26]

$$\frac{d\zeta_i}{dx} = \frac{1}{T}\left[\frac{\Lambda_{Ri}}{\Lambda_{Ri}^2 + \Lambda_{Ii}^2}\frac{d\Lambda_{Ri}}{dx} + \frac{\Lambda_{Ii}}{\Lambda_{Ri}^2 + \Lambda_{Ii}^2}\frac{d\Lambda_{Ii}}{dx}\right] \tag{3}$$

where Λ_{Ri} and Λ_{Ii} are the real and imaginary parts of the characteristic multipliers, which are the complex eigenvalues Λ of a real nonsymmetric matrix $[\Phi(2\pi)]$ called the *Floquet transition matrix* (FTM).[25] The FTM is obtained by integrating the equations of motion over one complete period of variation of the coefficients, and using a special set of initial conditions. The sensitivity of the ith characteristic multiplier Λ_i is given by[26]

$$\frac{d\Lambda_i}{dx} = \frac{d\Lambda_{Ri}}{dx} + i\frac{d\Lambda_{Ii}}{dx} = \frac{v_i^T \dfrac{d[\Phi(2\pi)]}{dx} w_i}{v_i^T w_i} \tag{4}$$

where v_i and w_i are, respectively, the left and right eigenvectors of the FTM $[\Phi(2\pi)]$ corresponding to the eigenvalue Λ_i. In general both v_i and w_i are complex-valued vectors. The FTM reflects a linearization of the rotor or coupled rotor–fuselage equations of motion about a trimmed flight condition. In general, a change in the design parameter x will also affect the FTM indirectly, i.e., through a change in the equilibrium position about which the equations are linearized. Therefore, if q is a vector of generalized coordinates defining the equilibrium position, the sensitivity of the FTM with respect to x will be given by[26]

$$\frac{d[\Phi(2\pi)]}{dx} = \frac{\partial[\Phi(2\pi)]}{\partial x} + \sum_{k=1}^{n_T}\frac{\partial[\Phi(2\pi)]}{\partial q_k}\frac{\partial q_k}{\partial x} \tag{5}$$

The first term in Eq. (5) is the sensitivity of the FTM. The second term represents the changes of the FTM caused by the changes in the equilibrium position associated with changes in x. This term is a direct effect of the nonlinearity of the aeroelastic problem and is expensive to obtain because it requires the sensitivity of the FTM to each of the trim variables q_K. This term may be small enough to be neglected,[26] but if it is not, then the semianalytic sensitivity of Eq. (5) can be more expensive than simple finite difference approximations. Setting up the calculations of all the sensitivities in Eq. (5) also requires that the internal coding of the aeroelastic analysis be modified rather extensively, and it is not a trivial task.

Problems in Calculation of Sensitivities of Inertia and Aerodynamic Loads

A practical obstacle to efficient sensitivity calculations is the need to modify the analysis computer programs. The underlying equations include portions modeling the structural, inertia, and aerodynamic loads acting on the system. The treatment of the structural portion is not very different from that needed in traditional structural optimization. In the most sophisticated models typically used the rotor blades are modeled as isotropic or composite beams undergoing moderately large elastic deflections, and the fuselages are modeled using traditional finite element techniques. The treatment of blade rotation is similar to that of column buckling, except for the direction of the loads. Customary structural optimization techniques are usually appropriate.

Problems begin to arise with the treatment of the inertia portion of the equations of motion, particularly as blade modeling becomes more sophisticated, and flight conditions and aircraft configurations become more complex. The inertia loads depend on the accelerations of each mass point of the system: the mathematical expressions for these accelerations can be so lengthy that it may not be convenient to include all of the terms in traditional mass and damping matrices, but it is necessary to leave many of them in a generic form as an external nonlinear forcing function.[1] This makes it very difficult to obtain efficient analytical or semianalytical sensitivities for the inertia loads.

The treatment of the aerodynamic terms can be even more complex. Many aerodynamic prediction issues are not completely understood, and a consensus on the best methodology has not yet been reached. The most sophisticated aerodynamic models include complex vortex-wake models, computational fluid dynamics-type models for the blade tips, and recursive or state-space formulations for the prediction of the unsteady aerodynamic coefficients of the blade airfoils. Intermediate sensitivity analyses for each of these portions will have to be obtained and coupled together before a global aerodynamic sensitivity analysis can be set up. The topic appears formidably complex, and very limited work appears to have been done in this direction in the rotorcraft community.

The treatment of the rotor wake can pose special problems. The most accurate rotor-wake prediction methods are those based on the free-wake approach, which allows the geometry of the vortex wake to evolve under the velocity field induced by the wake vortices themselves. Mathematically, a free-wake analysis usually takes the form of a double-nested loop.[27] The inner loop adjusts the wake geometry for a given distribution of blade circulation, and produces a converged geometry and inflow distribution. The outer loop adjusts the blade circulation until it becomes compatible with the geometry produced by the inner loop. Convergence of the double loop yields the aerodynamic loads for given blade pitch settings and blade motions; this is a key ingredient for the calculation of, for example, hub vibratory loads and blade stresses. If hub loads and blade stresses are required for a steady flight condition, then the double-wake loop needs to be placed inside a third loop that adjusts pitch settings and blade motion until a trim condition is reached.

Therefore, the analysis consists of a triple-nested loop, and this has important implications if finite difference-based sensitivities are desired. In fact, each of the nested loops must converge, and must do so with an accuracy that is consistent with the step size used in the finite difference approximation. In other words, if the step size is such that the perturbation of the behavior quantity of interest is of 0.1%, each of the inner loops must converge to within, say, 0.001%, with the convergence requirement becoming tighter and tighter going from the outer to the inner loops. These may not be trivial conditions to meet, as some widely used free-wake methods are known not to converge in some flight conditions (see discussion in Ref. 27) and are stopped after a predefined number of iterations. If the free wake does not converge, the sensitivities of the inflow distribution calculated using finite difference approximations can be meaningless. This example, and others not reported here for reasons of space, indicates that the step size for finite difference-based sensitivities can be an important practical problem. Unfortunately, little or no information is currently available on the extent of this problem because most optimization studies avoid free-wake calculations. Relatively large perturbation sizes may have to be used to absorb convergence problems when the analyses have a multiple nested-loop structure.

Availability of Suitable Approximate Analyses

Another key factor of the success of structural optimization has been the use of a series of techniques that are collectively known as *approximation concepts*. These techniques were pioneered by Schmit[28] and include 1) the construction of high-quality explicit approximations to objective function and behavior constraints, 2) design variable linking, and 3) temporary constraint deletion.[28]

Approximation of Objective Function and Constraints

The first technique replaces the original problem with a sequence of approximate, faster to solve, optimization problems; the sequence of solutions of the approximate problems converges to the solution of the original problem. The approximate problems can be obtained, for example, by expanding objective function and behavior constraints into first- or second-order Taylor series about the current design. This results in polynomial approximations to objective and constraints, and in an approximate optimization problem that can be solved quickly. The solution is a new design, about which objective and constraints are again expanded in Taylor series, giving an updated approximate optimization problem. This technique has been extensively used in helicopter problems. It is sometimes possible to increase the range of validity of the local approximations by performing the expansions in terms of intermediate, or intervening design variables. For example, a linear Taylor series expansion of a displacement constraint for a statically determinate beam is not very accurate if carried out in terms of cross-sectional dimensions, but is exact if reciprocal section properties are used instead.[20] Even if exact linearization cannot be achieved, a judicious use of intervening variables can reduce the nonlinearity of the problem.

No systematic attempt has been made to identify suitable intermediate variables in typical helicopter problems. For example, it is not known whether suitable combinations of blade cross-sectional dimensions or section properties could increase the linearity of constraints on aeroelastic stability, rotating and nonrotating hub loads, or blade stresses. Most optimization studies with these constraints, and which use approximations such as Taylor series expansions, require move limits on the design variables. For handling qualities optimization it has been shown[29] that a first-order Taylor series expansion in terms of selected stability derivatives can essentially linearize a constraint based on the moderate amplitude criteria of the ADS-33 Handling Qualities Specifications.[30]

Design Variable Linking

Design variable linking of some form is often applied in helicopter optimization. Linking reduces the number of design variables, which is especially important because most optimization studies include finite difference calculations of the sensitivities of some or all of the constraints. Helicopter optimization problems have rarely exceeded the number of 50 design variables, with most studies in the literature in the 10–30 variable range.

Temporary Constraint Deletion

The usefulness of constraint deletion varies greatly with the type of problem. For most optimization problems involving hub loads, blade stresses, and aeroelastic stability, the benefits are modest. It is true that in many problems only some harmonics and components of rotor loads and stresses appear in objective function or behavior constraints, and that only a few modes are potentially critical for aeroelastic stability. On the other hand, most of the computational effort often goes into the calculation of aerodynamic, structural, and inertia loads, which usually must be carried out even if only a few constraints are retained. For example, deleting aeroelastic stability constraints on the flap and torsion modes, which tend to be very stable, and retaining only those in lag, does not reduce the computational effort because the stability of all modes is obtained as a single procedure. Constraint deletion may be effective if the constraints are defined, for example, for different flight conditions. Then only the most critical flight conditions could be retained at any given step.

Applications of Design Optimization to Rotorcraft Problems

This section reviews some recent applications of design optimization to rotorcraft problems. The references will be divided into five groups: those with a primary orientation toward rotor dynamics, divided into 1) sensitivity analysis studies; 2) complete optimizations; 3) those with a different focus, such as handling qualities or preliminary design, or otherwise multidisciplinary; 4) those that, regardless of the disciplines covered, use some newer optimization technologies such as genetic algorithms or simulated annealing algorithms; and 5) those that report experimental evaluations of designs obtained from numerical optimization. This distinction is to some extent artificial, and therefore several references could have been legitimately included in more than one group.

Sensitivity Analyses

Semianalytical expressions for hub loads sensitivity have been presented by Lim and Chopra.[31] In Ref. 31, the aeroelastic analysis is based on a finite element method in time, and therefore the hub loads arise from the solution of a system of nonlinear algebraic equations, both in hover and in forward flight. The sensitivities are obtained from this system using chain-rule differentiation. Another methodology for hub loads sensitivity has been presented by Spence and Celi.[32] In this study the aeroelastic analysis is based on a quasilinearization method. The various algebraic expressions that make up the mathematical model are not expanded symbolically, but are assembled numerically as part of the solution process. This "numerical" formulation of the equations of motion is ideally suited for the derivation of semianalytical sensitivities using chain-rule differentiation.

The calculation of the sensitivities of the Floquet stability eigenvalues has been addressed by several studies in the period covered by the present article. Lim and Chopra provided the first such method,[33] based on a semianalytical approach. Expressions for the eigenvalue sensitivities are derived by chain-rule differentiation of the basic equations describing the Floquet stability analysis. These expressions contain the sensitivities of the element matrices, which are calculated analytically. The same method was later derived with a different approach by Shih et al.,[26] who also included nonlinear terms caused by the change in equilibrium position about which the aeroelastic system is linearized [see Eq. (4)]. It was shown that these terms, which are expensive to compute, have only a small effect on the sensitivity. Reference 26 also showed that a finite difference calculation of the eigenvalue sensitivities may yield incorrect results if some eigenvalues occur in closely spaced clusters.

Lu and Murthy[34] performed a modal coordinate transformation, based on the complex eigenvectors of the homogeneous system. Thanks to this transformation, the sensitivities with respect to each design parameters are calculated independently, instead of in a coupled way, as in Refs. 33 and 26. Another semianalytical method was presented by Venkatesan et al.[35] The method is formulated for the case of hover, which is governed by stability equations with constant coefficients. The eigenvalue sensitivity equations contain the sensitivities of the element matrices, which are calculated using finite difference approximations. For all of the stability sensitivity methods outlined earlier, the aerodynamic model is limited to quasisteady aerodynamics and uniform or linearly varying inflow. Except for Ref. 26, the inertia model is limited to a hub that is fixed or undergoes small-amplitude motions. Reference 36 takes the same analysis methodology of Ref. 35 as a starting point. Despite its title, however, it does not offer any real details on the stability sensitivity methodology.

A completely different approach has been proposed by Walsh and Young,[37] based on the use of the automatic differentiation code ADIFOR.[38] This code transforms any computer program into a new program that also computes sensitivities of given dependent variables with respect to given independent variables. The method has some similarities with the differentiation generated by symbolic manipulation software, but it operates directly on Fortran code. In Ref. 37, ADIFOR is used with the CAMRAD/JA[39] code. The code size increased from about 83,500 to about 198,000 lines. The sensitivities calculated by the resulting code agreed very well with those calculated using finite differences, and being essentially analytical derivatives, did not require the selection of a finite difference step size. The CPU time *increased* compared with finite difference sensitivities; this was attributed to the treatment by ADIFOR of the free-wake portions of the code. More recently, ADIFOR has been applied to a version of the TECH-01 comprehensive analysis code.[3] An unpublished report on this study[40] confirms excellent agreement of the sensitivities with those calculated using finite difference approximations, and shows results of a successful optimization carried out with the resulting code. The CPU time was about six times that required for finite difference calculations; this was tentatively attributed to the large size of the resulting code (the original analysis program consisted of over 140,000 lines).

When the solution process is iterative, such as for the solution of a system of nonlinear algebraic equations, ADIFOR generates a code that merges the iteration for the baseline solution with an iteration for the sensitivities. It has been shown that, for a large class of problems, if the baseline solution converges the derivatives are also likely to converge.[41] However, the convergence of the derivatives may be slower than that of the baseline solution; for a group of test problems the ADIFOR-differentiated algorithm required two to five times more CPU time than with finite differences.[41] This may help explain the increase in CPU times observed in the rotorcraft studies mentioned before. Automatic differentiation can become a very useful tool in helicopter optimization. The differentiation of entire comprehensive analyses is probably too ambitious at this time. On the other hand, ADIFOR-type codes could efficiently generate the sensitivities of *portions* of the mathematical model, and complement manual derivations and implementations of design-oriented analyses.

Aeroelastic and Dynamic Optimizations

Applications to Helicopters

A series of aeroelastic optimization studies was carried out by Ganguli and Chopra.[22,42,43] In Refs. 22 and 42 the purpose of the optimization was to reduce vibratory loads and dynamic stresses by tailoring the structural couplings induced by the composite rotor blade spar. Three different combinations of hub loads were used as objective functions. The design variables were the ply angles for several different one-cell[42] and two-cell[22,43] box-beam blade spars. Constraints were placed on the aeroelastic stability and on the blade natural frequencies. The gradients of objective function and constraints were computed using a semianalytical method, more efficient than finite difference approximations. The optimizer was the feasible direction-based code CONMIN.[44] The results indicate that significant reductions in vibratory loads and blades stresses can be obtained through the intermodal couplings that are generated by the laminate ply angles that are chosen by the optimizer. The optimization was carried out at one value of the advance ratio μ, i.e., $\mu = 0.3$, but load and stress reductions were observed for the optimized design also at all the other advance ratios considered ($\mu = 0.15-0.4$). A major contribution of these studies was that the optimization was repeated with a refined aerodynamic model including a free-wake model. In this case the aerodynamic portion of the sensitivities was computed using finite differences. The wake geometry was kept constant during the sensitivity calculations, which probably removed the numerical problems mentioned in a previous section of the present paper, and associated with the double nested circulation/geometry loop and with the problem of difficult convergence of the wake model used in the study. No convergence problems or multiple solutions were reported in these studies.

Yuan and Friedmann[23,45,46] carried out a structural optimization study, for vibration reduction, of a composite rotor blade with a swept tip. The objective function was a weighted average of the *N*/rev oscillatory hub load components. The ply orientations in the horizontal and vertical walls of the composite blade cross section, tip sweep, and anhedral were the design variables. Frequency placement and hover aeroelastic stability constraints were placed on the design. Objective function and behavior constraints were expanded in linear Taylor series expansion, with the latter using a mixed, conservative approximation[20]; semianalytical sensitivities were used.[35] The optimizer was the feasible direction-based code DOT.[47] The aeroelastic analysis was based on a moderate deflection finite element model, with special provisions for the modeling of the swept tip and of arbitrary, anisotropic cross sections; uniform inflow and quasisteady airfoil aerodynamics were used. Optimization proved very effective at reducing vibratory loads. A parametric study in Ref. 45 showed that the hub shears and moments, plotted as a function of the ply orientation of the horizontal wall, had local minima and extensive flat areas. Hover aeroelastic stability also showed local minima. No such complications appeared for the loads as a function of tip sweep. The initial designs were typically selected based on the results of this parametric study; this is the most likely reason why no convergence problems or multiple minima were reported.

Peters and coworkers also addressed rotor dynamics optimization. In Ref. 48, the objective was weight minimization subject to frequency placement constraints (up to 30), and to a lower bound on autorotation inertia. The design variables were cross-sectional dimensions, some blade geometry parameters, and two fiber orientation variables. The composite blade had an anisotropic crosssection. One interesting result of the study was the presence of local minima, which were strongly dependent on the details of the frequency placement constraint set. This confirms the results of the parametric study of Ref. 45.

A state-space representation of unsteady wake dynamics is the finite state wake model, developed by He and Peters. They also developed analytical sensitivities for this model,[24] coupled it with sensitivity equations for the remainder of the aeroelastic model, and used it in a study for the minimization of a weighted average hub vibratory loads and required rotor power. This is an extension of a previous study,[49] based on a less sophisticated aerodynamic model. A total of 63 design variables were used, describing spanwise distributions of blade chord and twist, flange and web thicknesses of the box beam-shaped structural cross section, internal lumped masses, and leading-edge weights. The behavior constraints were placed on rotor autorotational inertia, solidity, chordwise position of the center of mass, and placement of natural frequencies. CONMIN[44] was used to perform the optimization.

Rotor dynamics optimization studies have been performed by Chattopadhyay et al.,[50] Chattopadhyay and McCarthy, [51,53,54] and Chattopadhyay and Chiu.[52] The objective was a weighted average of blade weight and *N*/rev vertical shear,[50] or of selected vibratory load components.[51,54] Design variables were, depending on the study, blade stiffnesses or cross-sectional dimensions of an internal box beam spar, taper ratio, root chord, root mass radius of gyration, twist, and nonstructural weights. In Ref. 50 constraints were placed on autorotational inertia, blade frequency placement, and centrifugal stresses. In Ref. 51 constraints on blade weight, rotor thrust, and components of vibratory loads not included in the objective function were added. In Ref. 54 an additional constraint was placed on the chordwise position of the section center of mass, to guard against aeroelastic instabilities. The optimizer was CONMIN,[44] the analysis typically CAMRAD[55]; objective function and constraints were expanded in first-order Taylor series with finite difference gradients.

A multilevel decomposition approach was followed in Ref. 56. A three-level procedure was used. Level 1 was an aerodynamic performance optimization, with the power coefficient as the objective function, and spanwise chord and twist distributions as design variables. In level 2 some components of the vibratory hub loads were minimized; the constraints included autorotational inertia and aeroelastic stability; the spanwise flap and lag stiffness distributions and the values of nonstructural weights were the design variables. Level 3 was a weight minimization using the cross-sectional dimensions of a box beam spar as design variables. Through the use of constraints, level 3 implemented a design recovery phase that attempted to match the stiffnesses obtained from level 2.

The research conducted at NASA Langley Research Center on multidisciplinary helicopter optimization has been described in several papers including, for the period covered by the present survey, Refs. 14 and 57–59. An integrated aerodynamic/dynamic optimization procedure for rotor blade optimization is presented by Walsh et al.[60] This study extended a previous one on performance optimization alone.[61] The design variables defined chord and twist distribution of the blade, its bending and torsion stiffness distribution and size, and the spanwise location of three tuning masses. The design was for a 1/6th-scale rotor model. Three flight conditions were considered, namely hover, forward flight, and a maneuver simulated by increasing the rotor thrust. Behavior constraints were placed on 1) performance, enforcing upper bounds on required power and airfoil drag coefficients around the azimuth, requiring that the rotor be trimmable, and placing a lower bound on the blade chord; 2) dynamics, with upper bounds on blade weight and lower bounds on autorotational inertia, and constraints on the placement of natural frequencies. The objective function was a weighted average of the powers required in the three flight conditions and of the *N*/rev vertical hub shear. Aeroelastic stability was not considered in the optimization. The optimization was carried out as a sequence of linear approximate problems with the move limits on the design variables. The aeroelastic analysis for forward flight and maneuvers was CAMRAD/JA,[39] used with a uniform inflow model. The optimizer was CONMIN.[44]

Two design procedures were considered. In the first, a performance optimization was performed first, with powers only in the objective functions, blade geometry as design variables, and performance constraints; a dynamics optimization was performed next, based on the remainder of objective, design variable, and constraints. The second design procedure was an integrated optimization that solves the performance and dynamics portions simultaneously; in this case there were 19 design variables and 80 behavior constraints. The optimized designs were feasible; the one obtained from the integrated optimization appeared slightly superior. It is interesting that the optimum designs were not too different from a reference blade optimized without the use of formal numerical optimization techniques.

In Ref. 62, the analysis was replaced by a neural network. In Refs. 63 and 64, structural optimization was added to the formulation of Ref. 60, and the integrated problem was solved using a two-level decomposition methodology conceptually based on those of Refs. 65 and 66. In Ref. 64, the upper level optimization adjusted blade planform, stiffnesses, and tuning masses to minimize a weighted average of the powers required in three flight conditions and of the *N*/rev vertical hub shears in forward flight, subject to the same constraints as in Ref. 60 plus upper bounds on the average axial strains. The lower level adjusted the dimensions of the cross section (wall thicknesses and lumped areas representing longitudinal stringers) to minimize the difference between the stiffnesses corresponding to these dimensions and those required by the upper level optimization, i.e.,[64]

$$F = \left[\frac{EI_{zz} - (EI_{zz})^*}{(EI_{zz})^*}\right]^2 + \left[\frac{EI_{xx} - (EI_{xx})^*}{(EI_{xx})^*}\right]^2 + \left[\frac{GJ - (GJ)^*}{(GJ)^*}\right]^2 \quad (6)$$

The coordination between upper and lower level was implemented by adding a special constraint to the upper level problem, namely,

$$g = F_U - (1 + \varepsilon)F_0^L \leq 0 \quad (7)$$

where F_0^L is the optimum of F, Eq. (6), at every cycle, F^U is an estimate of the change in F_0^L that would be caused by a given change in the upper level design variables, and ε is a specified tolerance. This constraint penalizes the upper level if it generates stiffnesses that the lower level cannot match and, which, therefore, could not be obtained in practice. CONMIN[44] was used to solve both upper and lower level. The two-level optimization proved efficient and more robust than the single-level one, because it managed to generate feasible designs from an infeasible starting point even if the single-level version did not. The lower-level optimization problem was reformulated in Ref. 67, using a response surface methodology; in this case, the objective function of the lower-level problem was replaced by a quadratic Taylor series in terms of the upper-level design variables.

References 68 and 69 describe an optimization for vibration reduction. The design variables defined the spanwise mass and stiffness distribution, structural twist, and tip sweep, for a total of up to 30 design variables. Several combinations of vibratory loads at the hub and the pilot seat were used as objective functions. The analysis was CAMRAD/JA,[39] used with uniform inflow for improved efficiency; however, a validation with a free-wake model was also carried out. The optimizer was CONMIN.[44] Vibration minimization at the hub did not necessarily result in vibration minimization at the pilot seat. Using uniform inflow produced vibration reductions that often were optimistic, compared with those based on the free-wake analyses. It was also noted that "the optimization program is only as good as its core analysis model. Any deficiencies therein will certainly be reflected in the optimization process."[69]

Applications to Tilt-Rotor Aircraft

Research on the multidisciplinary optimization of rotor and wing of a tilt-rotor aircraft has been described by Chattopadhyay and her coworkers in a series of papers. Typical design variables defined the chord, twist, and sweep distribution of the rotor blades, the thickness distribution of the cross section of the wing box beam, and wing root chord and taper ratio; side constraints were placed on some design variables. One,[70,71] two,[75] or three[72,73] flight conditions were considered. The behavior constraints fixed the hover and cruise thrusts and power during the optimization, placed lower bounds on the lowest-frequency rotor mode and on the autorotational inertia of the rotor, upper bounds on the blade weight, lower bound on high-speed aeroelastic stability, and upper bounds on the wing root stresses (the specific constraints vary slightly in the different references). In Ref. 70, the objective function was the propulsive efficiency in axial flight. In Ref. 71, it was the weight of the box beam placed inside the rotor blade. In Ref. 74, the objective functions attempted to maximize the propulsion efficiency and minimize total weight. In Ref. 75, it was the weight of engine, transmission, and fuel, obtained from preliminary design-type sizing equations. In Refs. 72–74, several constraints were absorbed into the objective functions, which in turn were collapsed into a single objective function using the Kreisselmeier–Steinhauser (KS) function,[20] so that the optimization became unconstrained. The gradients were calculated using finite differences. The optimization was carried out as a sequence of approximate problems obtained using linear Taylor series expansions,[70] or a two-point exponential approximation[76] with move limits on the design variables.[71–75] The optimizer was CONMIN.[44] The analysis was CAMRAD,[55] used with uniform inflow and quasisteady aerodynamics,[71,73,74] or CAMRAD/JA.[70,75] In Ref. 72 a simpler analysis limited to aerodynamic performance was used, intended as the top level of a possible multilevel approach to the optimization. Subsequent references describe the multilevel formulation of the problem; because the lower level subproblems are solved using simulated annealing, these references are reviewed in a later section on Emerging Optimization Technologies.

Additional Helicopter Applications of Optimization

Flight Dynamics

An interactive optimization software facility has been developed by Tischler et al.[77] The code addresses the design of flight-control systems, formulated as a multiobjective optimization problem. The design variables are the gains and other parameters of the control system. The constraints originate from handling-qualities specifications. For helicopter applications, the ADS-33 criteria[30] are used. A sequential quadratic programming algorithm is used. The objective function is formulated in minimax form

$$F(X) = \min[\max \alpha_i f_i(X)], \qquad 1 \leq i \leq m, \alpha_i \geq 0 \quad (8)$$

where X is the vector of design variables, and $f_i(X)$ is the ith objective function; the α_i are weights that can be changed interactively to determine a tradeoff among the various objectives. Each of the functions $f_i(X)$ can be a performance criterion to minimize or a constraint to satisfy. With this form of objective function, the optimizer concentrates on one objective function at a time, i.e., the least satisfactory performance criterion or the most violated constraint, with the choice also influenced by the weights α_j. A sophisticated graphical user interface supports the decisions of the analyst. The aircraft is modeled as a linearized system of differential equations, which is derived outside the code itself.

Unpublished reports indicate that local minima have been encountered. The constraints can be implemented as *hard* or *soft*, depending on whether they must definitely be satisfied or are desirable values that the optimizer should try to achieve. They can be adjusted interactively and, together with the weights α_j in the objective function [Eq (8)], give the designer flexibility in carrying out the optimization. The optimization program acts as a designer's assistant; the designer interactively adjusts the problem and path taken to reach the optimum. The concept of having the human designer share the optimization loop with traditional mathematical programming methods is an important contribution of this research. It can be argued that this approach denies an important benefit claimed for numerical optimization, namely that the optimization algorithm does not carry the same biases of the designer and, therefore, it is free to find solutions that the designer might not have conceived.[78] On the other hand, many practical design problems do have some uncertainty in their formulation. For example, a constraint might be a result of a requirement that can be waived or relaxed by the customer, and therefore a flexible formulation such as that of Ref. 77 can more accurately represent a practical design situation.

A more traditional approach has been followed in Refs. 29, 79, and 80, which describe the multidisciplinary optimization of rotor and flight-control systems with aeroelastic stability and handling quality constraints. The handling quality constraints enforced a subset of the specifications of ADS-33,[30] including constraints on pole placement, on the shape of selected frequency responses to pilot inputs, and on the shape of the time histories of the response to selected pilot inputs. The design variables were the flap and lag stiffnesses of the rotor and the flap–lag elastic coupling factor, plus control system gains and other parameters of the feedforward loop. The objective function was a weighted average of the swashplate control displacements and rates for two preassigned maneuvers. The optimization was carried out at one velocity, or advance ratio, and at two velocities simultaneously. In the latter case, the gains of the control system at each speed were separate design variables, and all of the constraints were separately imposed at each speed. The maximum number of design variables and constraints was 15 and 48, respectively. A sequential quadratic programming algorithm was used to solve the problem. The gradients of objective function and behavior constraints were calculated using finite difference approximations. The analysis was based on a coupled rotor–fuselage model with rigid blades with offset hinges and root springs.

The study concluded that designing rotor and flight-control system simultaneously, rather than the rotor first and the control system next, may lead to designs that require lower control effort. Two problems were identified in Ref. 79, namely the large computer time required, of the order of 10–20 h on typical workstations, and the lack of robustness of the design, which would become infeasible at speeds other than those used in the optimization. The first problem is mostly because of the constraints on the time histories of the response to pilot inputs, and was addressed in Ref. 29. The gradients of two such constraints were calculated from the time histories of approximate, low-order linearized models that matched the frequency response of the full-size linearized model over a preassigned frequency range. The technique reduced the additional cost of calculating these gradients by two orders of magnitude. The resulting optimization achieved good reductions of the objective function, although not as large as with the original full nonlinear gradients. The second problem identified in Ref. 79 was addressed in Ref. 80, namely the infeasibility in off-design conditions, by adding a robustness constraint. A robust performance constraint proved too stringent, and a relaxed stability constraint had to be implemented instead. The robustness of the design was greatly enhanced, and all of the off-design constraint violations were greatly reduced or eliminated.

Aerodynamics

Reference 81 outlines the use of optimization at Aerospatiale (now Eurocopter France) and ONERA around 1990. The optimization was primarily aerodynamic: the design variables defined the airfoil shape, the objective function was the c_x coefficient at a Mach number $M = 0.77$ (advancing blade), the constraints included upper bounds on the absolute value of the pitching moment coefficient at $M = 0.77$ and lower bounds on the thickness-to-chord ratio. The optimizer used was CONMIN.[44] A multidisciplinary flavor was obtained by moving from airfoil to blade design, and by adding performance and load considerations. The design variables described the blade geometry and the spanwise distribution of the airfoils. A weighted average of the power required at moderate and high speed was minimized. Reductions of 3–5% in required power were reported.[81] Additional details of the airfoil design optimization are provided in Refs. 82 and 83. This work evolved in the optimization program ORPHEE, an application of which is described in Ref. 84; although the underlying aeroelastic code was capable of more sophisticated modeling, rigid blades and uniform inflow were used in the study, which focused primarily on aerodynamics and airfoil design.

Another aerodynamic/performance optimization capability is described in Refs. 85 and 86; the study addresses the aerodynamic optimization of a variable diameter tilt rotor in hover and cruise flight. The analysis consisted of a hover free-wake model based on an iterative relaxation scheme. The sensitivities were obtained using finite difference approximations, starting from the baseline solution, perturbing each design variable in turn, and letting the relaxation continue until convergence was achieved again. Reconvergence was achieved very rapidly. The optimization was carried out using a sequential linear programming technique. A total of 48 design variables were used, namely chord, twist, and sweep at 16 radial stations. Improvements in hover and cruise efficiency could be obtained without complex variations of planform geometry.

Preliminary and Conceptual Design

In Ref. 87, Hajek outlines the use of optimization in the preliminary design phase at MBB Helicopter Division (now Eurocopter Deutschland) around 1990. The disciplines involved were aerodynamics, mass estimation, engine modeling, and cost modeling. Vibrations, acoustics, loads, and handling qualities were not considered. In general terms, the design variables were mission and empty weight, rotor size, cabin size, engine power and transmission rating, and several measures of performance. The analysis was based on semiempirical or empirical relationships, or very simple theoretical analyses. Objective functions included power required, fuel consumption, and a productivity parameter related to payload, empty weight, and cruise speed. No precise information on the solution algorithm was provided; the technique appears to be sequential linear programming. No illustrative results are presented.

Finally, optimization has been applied to the conceptual design of an advanced civil tiltrotor by Schleicher.[88] The study was conducted using a helicopter-sizing code, coupled with a Fletcher–Reeves-type optimizer. This suggests that the optimization was unconstrained; however, no further details on the formulation of the optimization problem are provided.

Emerging Optimization Technologies

Genetic Algorithms

Recently, there has been a growing interest in genetic algorithms (GA) for helicopter multidisciplinary optimization. A series of papers[89–92] explored the use for rotor blade design. In these algorithms, the design variables are expressed as strings of 0s and 1s; these bits can be the exponents of 2 in a binary representation, or denote with a particular combination one of a number of possible discrete values for the variable, much as a pointer to a look-up table would. Obviously, for continuous variables, the longer the binary string, the more precise the value of the variable. The strings corresponding to each variable are then joined together to form a longer binary string that defines the entire design.[20] For a 42-design variable example presented in Ref. 92, and a relatively coarse precision, this longer design string was composed of 179 zeros and ones. The GA starts with a certain number of initial design strings with random combinations of 0s and 1s. The algorithm then proceeds by generating new designs with bit operations devised to mimic mathematically some biological evolutionary phenomena, such as mating, crossover, and mutation. Through these operations the design evolves toward the optimum.

GA can easily deal with integer and discrete variables. Additionally, they have a better likelihood to identify global minima than conventional gradient-based algorithms. In fact, if the initial designs span a sufficiently large portion of the design space, subsequent designs may cluster in groups around different local minima, one of which might be the actual global minimum. A third advantage is that they can be easily implemented on parallel computers, because the evolutionary steps may be applied to independent pairs of designs. The main drawbacks of GA are that the size of the problem increases rapidly if many design variables need to be determined precisely (and therefore must be represented with long binary strings), that convergence can be slow because the properties of the feasible region are not exploited directly, and that intermediate designs may be infeasible.

A basic GA algorithm is proposed in Ref. 89; multistage searches are proposed to increase efficiency. This means that the optimization is repeated several times, starting from coarser representations (which require shorter design strings), and progressively increasing the precision of the formulation. In subsequent papers,[90-92] the optimization is decomposed into subproblems, with a structure in which the subproblems optimization represents an inner loop, and the system-level optimization an outer loop. The subproblems are identified by a neural network, which determines the relationships between inputs (design variables) and outputs (objective and constraint values) and, therefore, which groups of design variables and objective/constraints are most closely coupled. In Ref. 92, the objective function was a weighted average of vibratory hub loads. The constraints covered the power required in hover and forward flight, the figure of merit of the main rotor, an autorotation index related to blade mass, weight, and failure stresses. The design variables included blade geometric quantities and the cross-sectional dimensions of the blade spar, assumed to vary in discrete steps to simulate the number of layer of a composite laminate. The 42 design variables were represented with a total of 179 bits, for a precision of about two significant digits. A multibody dynamic model was used for the rotor blade, together with an unsteady aerodynamic model. The training of the neural network used to decompose the problem required from 1450 to 12,800 function evaluations, depending on the neural network algorithm and the desired accuracy. After 70 system level optimizations a reasonable convergence was obtained.

A number of additional optimization studies using GA have been carried out. In Ref. 93, the number of blades, airfoil sections, solidity, twist, tip speed, disk loading, and taper were the design variables for an acoustic optimization study. The design string length was 27 bits. A more recent study[94] focuses on the acoustic design of tilt-rotors using GA. In Ref. 95, the objective was to design a rotor airfoil. The blade was assumed to be *frozen* at the azimuth angles of 90, 180, and 270 deg, and was analyzed with a fixed-wing type, steady aerodynamic model. Structural constraints were simulated by placing lower bounds on the airfoil thickness. The focus was on conceptual design in Refs. 95–98, in which the objective was power and weight minimization subject to a variety of performance and mission constraints. Two integer, two discrete, and five continuous design variables resulted in a 28-bit design string. A typical industry conceptual design code was used for the analysis. From about 1500 to about 4500 function evaluations were needed for convergence. The design variables included truly discrete variables such as the number of rotors and rotor blades, the presence or absence of a wing and a propeller, and the number of engines. Therefore, this is clearly an optimization problem that would be impossible to solve with gradient-based optimization methods. Rotor design for minimum weight and power required were studied in Refs. 99 and 100 using the same design variables of Ref. 93. Upper bounds on local aerodynamic angles of attack and Mach number at 90 and 270 deg, and a lower bound on the thrust were appended to the objective functions using a quadratic exterior penalty function. In Ref. 101, the same constraints and objective functions are collapsed into a single objective function using the KS function.

The problem of Ref. 92 could be solved with conventional gradient-based methods. It would be interesting to compare conventional solutions with those obtained with GA, to assess advantages and disadvantages of this technique. GA require a large amount of function evaluations, and can provide only low or moderate precision unless lengthy design strings are used, which would further increase the size of the optimization problem. Therefore, they only seem suitable for small problems and for problems in which limited accuracy is required, and in both cases conventional gradient-based techniques are likely to be much more efficient. On the other hand, if the feasible region for a given problem is really heavily nonconvex or even disjoint, and if objective function and constraint gradients must be calculated using finite differences, then conventional techniques may indeed turn out to be more expensive. In fact, in this case it would be prudent to repeat the optimization starting from different designs, to increase the likelihood of locating a global minimum. The best approach might be to perform an initial search with GA, or some other probabilistic search method, select some promising configurations, and then optimize these further using conventional gradient-based methods.

Simulated Annealing

Another emerging optimization technique is the simulated annealing algorithm, which mimics mathematically some physical processes that occur in the solidification of metals and the formation of crystals.[20] Like GA, simulated annealing is a probabilistic design method, does not require derivatives, can deal with integer or discrete variables and with nonconvex design spaces, and for problems with local minima can increase the likelihood of finding a global minimum. Also like GA, it can be much more expensive than conventional gradient-based methods when the properties just mentioned are not required in the optimization. Simulated annealing has been applied by Chattopadhyay and Seeley[102] to high-speed prop-rotor design. The algorithm is composed of the following steps[102]:

1) The current design is X, the corresponding value of the objective function is $F(X)$

2) Perturb the current design: the perturbed design is X_{new}.

3) If $F(X_{\text{new}}) \leq F(X)$, then X_{new} becomes the current design because it is as good as or better than X.

4) If $F(X_{\text{new}}) > F(X)$, then X_{new} is a worse design, but accept it anyway if a random number P, with $0 \leq P \leq 1$, drawn for this design is greater than an acceptance probability P_{acc} defined as

$$P_{\text{acc}} = \exp(-1/T) \tag{9}$$

5) Repeat the previous steps until convergence; reduce T as the algorithm proceeds.

Accepting a design, even if it is not an improvement, allows the algorithm to jump out of a local minimum, which a gradient-based algorithm usually would not be able to do. As T in Eq. (9) decreases, so does the probability P_{acc} of accepting a worse design, and the method settles in the search of the minimum. No information on the shape of the feasible region is used, which causes the method to be less efficient than gradient-based algorithms. Because the design can be perturbed with arbitrary methods, the design variables need not be continuous, and discrete or integer variables can be included in the optimization.

Simulated annealing was also used by Refs. 103–105 in the lower level subproblem of a multilevel optimization approach to prop-rotor design. In Ref. 103, the upper level was an aerodynamic performance optimization problem, in which the design variables govern the blade geometry. The lower level was a structural optimization problem, in which the design variables were the ply angles of the box beam spar of the blade. It was assumed that the ply angles could only take discrete values, which justified the use of simulated annealing. Refs. 104 and 105 extended the work of Ref. 103 to include takeoff performance in the formulation of the upper-level optimization problem.

Experimental Verifications

References 106 and 107 present the results of the optimization of a four-bladed articulated rotor for vibration minimization. The design variables are the spanwise mass and stiffness distribution. The objective function is a weighted average of the hub loads in the fixed system. Some constraints on the static and dynamic behavior of the blade are appended to the objective function. In Ref. 106, only N/rev harmonics were considered in the optimization, in Ref. 107, both N/rev and 2 N/rev were considered. Other constraints ensure that the blades can actually be manufactured. The airloads were held constant

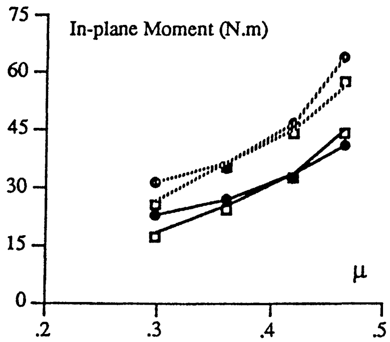

Fig. 2 Comparison between the 3/rev in-plane moments of the optimized (——) and reference (- - -) blades. □ and ○, calculations; and ■ and ●, tests. Figure taken from Ref. 108.

during the optimization. Predicted and actual vibratory loads of the optimized rotor are not compared in the paper, and therefore it is impossible to determine the role played by possible inaccuracies of the analysis. An experimental comparison between the loads of the baseline and of the optimized, Mach-scaled model rotor shows a reduction of 4/rev hub loads from 45 to 77%, depending on the specific load component and flight speed[106]; further reductions could be obtained by including the $2N$/rev components.[107]

Another optimization study with experimental validation has been presented by Leconte and Geoffroy.[108] The objective was to minimize the moment caused by 3/rev in-plane shears for a four-bladed articulated rotor, using the spanwise mass and stiffness distribution and weight and position of two nonstructural masses as design variables. A finite element model for the anisotropic composite cross section was used, with three-dimensional aerodynamics and quasisteady airfoil characteristics. The optimizer was CONMIN,[44] and finite difference gradients were used. Behavior constraints were placed on some 4/rev hub load components and frequency placement. The solution was sought at a specified value of thrust coefficient and advance ratio, but it was verified over a broader range of thrust and speed values. The results show a significant decrease of the 3/rev inplane moment; a vector plot of the contributions from all of the modes shows that this was achieved by reducing most magnitudes and altering all of the phases for each mode. Both the baseline and the optimized blades were built and tested in a wind tunnel. Figure 2 shows a comparison between the in-plane moments for the baseline and the optimized rotor; both the calculated and the experimental results are shown. The figure clearly indicates a reduction of the objective function in the optimized configuration at all advance ratios. The agreement between theoretical and experimental values appears to be reasonably good. Numerical robustness tests indicate that the solution is basically insensitive to variations of ±10% of the design variables at the optimum.

Tuning masses were used by Pritchard et al.[109] to reduce vibratory hub loads. Objective function and some of the behavior constraints were formulated to try to minimize the loads without incurring an excessive weight penalty. Magnitude and spanwise locations of the tuning masses were the design variables. Additional behavior constraints placed upper and lower bounds on the blade natural frequencies. The code CAMRAD/JA[39] was used; objective function and behavior constraints were expanded in linear Taylor series, and the resulting sequential linear programming problem was solved using CONMIN.[44] The theoretical results were compared with tests of a Mach-scaled, 1/6th-scale four-bladed UH-60 Blackhawk rotor, in which the vibratory loads were measured for varying spanwise positions of one blade tuning mass. It is interesting to note that the vibratory loads for the baseline and the optimized rotor were not predicted very accurately at any of the advance ratios: the underprediction ranged from a factor of 2 to about 5. Nevertheless, predicted and actual spanwise positions of the tuning mass for minimum 4/rev hub shear were in reasonably good agreement. Furthermore, the ratios between the baseline and the optimum shears were predicted with less than 3% error.

An extensive experimental program to validate the predictions of a blade design optimization methodology was carried out in Refs. 110 and 111. This methodology is based on the minimization of vibration indices that include blade natural frequencies, generalized forcing functions, and generalized inertias of given blade modes, and which require very simple analyses for their calculation, i.e., not sophisticated aeroelastic or comprehensive analyses. These metrics are discussed in greater detail in Ref. 112. In Refs. 110 and 111, the optimization was performed with the ADS-1 code.[113] The tests were carried out on a family of 1/5th-scale, Froude-scaled rotors. The simple, modal-based metrics proved capable of reducing blade vibratory loads (frequency placement criteria alone, on the other hand, could not reduce loads consistently). Figure 3, taken from Ref. 112, shows the magnitude and phase of the various modal contribution to the total 4/rev vertical shear and hub moments. The figure indicates that the reduction in vibratory loads comes from the reduction in magnitude of all the

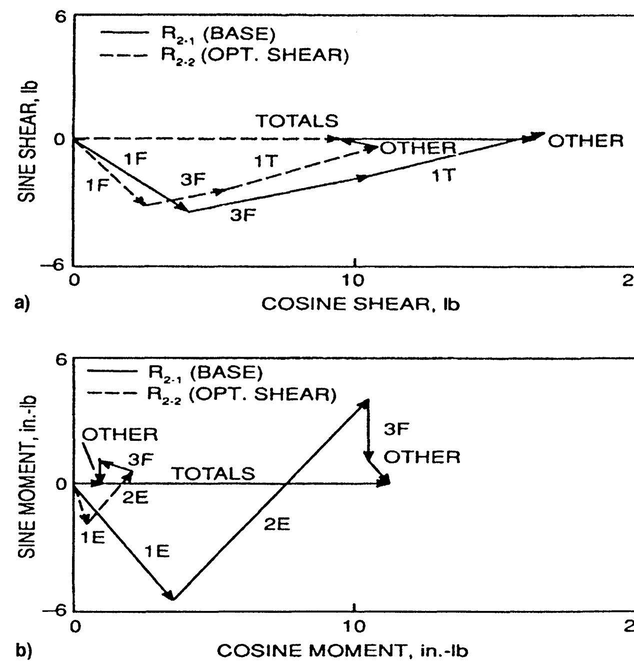

Fig. 3 Comparison of 4/rev modal contributions for baseline (——) and optimized (- - -) blade, μ = 0.4; 1E: first edgewise mode, 1F: first flatwise mode, 1T: first torsion mode, etc. Figure taken from Ref. 112: a) 4P vertical shear and b) 4P yawing moment.

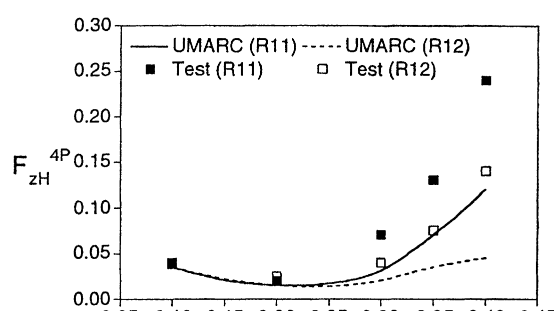

Fig. 4 Vertical 4/rev hub shear for baseline (R11) and optimized (R12) blades as a function of advance ratio μ. Figure taken from Ref. 114.

components, rather than from phase shifts that reduce the vector sum of the components without reducing their magnitude.

The same designs have been recently studied with a comprehensive analysis code coupled to an optimizer by Ganguli et al.[114] Quasisteady aerodynamics and linear inflow modeling underpredicted the vibratory loads significantly; free-wake modeling was judged critical for hub loads predictions. Figure 4 compares analytic predictions and experimental results for the baseline and the optimized rotor. At medium- and high-advance ratios, the error of the analytical predictions is as large as the improvements brought about by the optimization. Nevertheless, the optimization does generate a design that has consistently lower hub shears than the baseline. Another interesting conclusion is that the modal-based approach can generate designs that are almost as good as those obtained with a much more sophisticated analysis, and with a much lower computational effort. However, Ref. 114 warns that this may be a result of the lack of strong aeroelastic interactions in a Froude-scaled model rotor: in Mach-scaled or full-scale rotors the modal-based procedure may not be as powerful.

Concluding Remarks

As this review demonstrates, there is considerable activity in applications of design optimization to helicopter problems. The accuracy of the underlying analyses remains an important problem because optimization results obtained using analyses of insufficient accuracy have only very limited value. However, as the reliability of predictive analytical tools continues to increase, the role of optimization in actual helicopter design will likely be expanded. In the meantime, one should inject some realism in evaluating the actual results of an optimization. Perhaps each design optimization study should include an evaluation of the robustness of the optimum, possibly based on optimum design sensitivity analysis methodologies,[20] to help determine how sensitive the optimum is to modeling inaccuracies.

No algorithmic silver bullet has emerged to make the solution of helicopter optimization problems as computationally efficient and robust as for traditional structural optimization problems. It is currently unrealistic to expect to find an optimum for the cost of 10–15 analyses. Efficiency can increase if the gradient information is calculated analytically or semianalytically rather than by finite differences, but this requires modification of the analysis computer programs, which may pose a roadblock in industrial applications. Efficiency could also increase with judicious use of approximation concepts, but additional research is needed to further specialize this technique to helicopter problems. Future advances in computer hardware could make these issues partially irrelevant, particularly if large-scale parallelism can be practically implemented. On the other hand, future research may show that accurate predictions require substantially more complex and time-consuming analyses than current ones. Therefore, there is still much value in continuing research in more efficient sensitivity calculations and approximation concepts.

The traditional gradient-based optimization methods will probably remain the algorithmic mainstay of helicopter design optimization for the foreseeable future. Emerging technologies such as simulated annealing and GA currently seem too inefficient for problems of realistic complexity. They may play a role in problems with truly discrete design variables, and could prove useful for a preliminary search of the design space, to be completed with traditional algorithms. Nonconvexity and local minima are considered traditional enemies by many practitioners of design optimization, and much research is directed toward algorithms with global convergence properties. It should be kept in mind, however, that local minima can represent alternate design solutions. In practice, these designs may prove better than the theoretical global optima, because of criteria not considered in the formulation of the original optimization problem. Future research should help determine when the nonconvexity of the feasible region is truly an intrinsic negative feature of the optimization problem to be tackled with globally convergent algorithms, when it is caused by a poorly chosen formulation of the problem, or when it simply reflects the presence of more than one acceptable design solution.

It is not easy to verify experimentally that a design obtained using optimization techniques is truly *the* optimum design. In fact, one would have to build and test not only the design corresponding to the theoretical optimum, but also enough designs in its neighborhood to verify that theoretical and actual optima are sufficiently close. It is easier to simply verify that the use of formal optimization techniques leads to a better design, because one only needs to build and test the baseline and the *optimum* design, and compare the two. This has been done in a few, very interesting studies in the area of rotor vibration reduction. Often there were some significant discrepancies between the predicted and the measured loads on the optimum designs. This confirms that several basic aspects of the problem are not yet fully understood. On the other hand, in all such studies, the designs did somehow improve through the use of formal optimization techniques, and this is very encouraging.

Researchers in helicopter applications of optimization face a complex multidisciplinary problem, with several possible choices of design variables, objective functions, behavior and side constraints, analysis models, sensitivity formulations, approximation concepts, and optimization algorithms, not to mention the many types of results that can be generated and presented. Therefore, the helicopter community should agree on a small number of representative test cases, augmented by judicious experimental verification, which researchers in the field should adopt. Studies such as those of Refs. 17 and 18 have been exceptionally useful because they have provided a realistic assessment of the state of the art, and clearly indicated where additional work was needed. Helicopter design optimization would greatly benefit from similar work that judiciously bridges the gap between theory and practice.

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